ST PAUL'S SCHOOL
JUNIOR SCHOLARSHIP EXAMINATION

MAY 2014

MATHEMATICS

2 hours

Answer as many questions as you can in any order you wish.

Question 11 should be answered on the insert sheet, and this should be handed in with the rest of your answers.

Credit will be given for reasoning and working where appropriate.

Give formulae for areas and volumes. When the answer is a fraction it should be given in mixed form, e.g. $3\frac{4}{5}$

The total number of marks for this paper is 109.
The mark allocation is shown in brackets at the end of each part of each question.

Please write your name on top of your answer sheets.

<u>CALCULATORS MAY NOT BE USED.</u>

1 Work out the values of the following, giving your answers as simplified fractions (mixed if appropriate).

    a) $\frac{2}{3} + \frac{3}{4} \times \frac{8}{15}$ [2]

    b) $\left(\frac{1}{8} - \frac{1}{11}\right) \div 1\frac{25}{44}$ [2]

2 Given $p = 8$ and $q = -2$, find the value of

    a) $pq^2(p - q)$ [2]

    b) $\dfrac{16 - q^2}{3p}$ [2]

3 Solve the following equations for x:

    a) $3x - 1 = 4 - 5x$ [2]

    b) $\dfrac{x - 7}{3} = \frac{1}{2}(27 - x)$ [2]

    c) $\dfrac{3x - 1}{x} = 7$ [2]

4 A brother and sister share out their collection of 5000 stamps in the ratio 5:3. The brother then shares his stamps with two friends in the ratio 3:1:1 and keeps the largest share for himself. How many stamps does each friend receive?   [2]

5

    a) Which of the following is the correct answer to $\dfrac{112.2 \times 75.9}{6.9 \times 5.1}$?

        Explain how you know.

        (i)   242   (ii)   20.4   (iii)   25.2   [2]

    b) Write down the correct answer to $\dfrac{1122 \times 759}{69 \times 0.00051}$.   [2]

6 Alan and Brenda are queuing at the ticket office. Alan is m places in front of Brenda. There are n people behind Alan and p people in front of Brenda. Find an expression for the number of people in the queue.   [2]

[if Alan was 3 places in front of Brenda it would mean that there were 2 people between Alan and Brenda]

7   a)   A car travelling at 90 km/h is 5 km behind another car travelling at 70 km/h in the same direction. How long will it take the first car to catch the second?   [3]

b)   Once the first car has caught the second, the slower car increases its speed by 20%. In order for the two cars to travel at the same speed, by what percentage should the faster car decrease his speed?   [3]

8   a)   Expand and simplify the following expressions

    (i) $(2x - 1)(5x + 2)$ [2]

    (ii) $(3x - y)(3x + y)$ [2]

b)   If $(2x - 5)(3x - 2)(ax + 3) = 12x^3 - 20x^2 + cx + b$,
find the values of *a, b* and *c*.   [3]

**9**   The prime factorisation of three numbers is shown below, where *a, b, c, d* and *e* are all prime numbers.

$$X = a^3 \times b^2 \times c^2 \times e$$
$$Y = a \times b^2 \times c \times d \times e$$
$$Z = a^2 \times c \times e$$

a)   Find the highest common factor of *X* and *Y*.   [1]

b)   Find the lowest common multiple of *Y* and *Z*.   [1]

c)   If $X = W \times Z$, what is *W*?   [1]

d)   When *X* is multiplied by a number *T*, the result is a square number. Write down the prime factorisation of the smallest possible value of *T*.   [1]

10   a)   A string of length 40 cm has its ends attached to the top two corners *A* and *B* of a picture so that it could be hung over a nail. Given that the top of the picture is 16 cm below the nail and the picture is level, how wide is the picture?   [3]
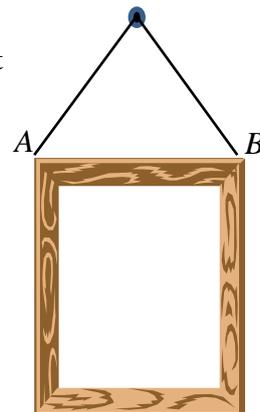
b)   I now remove the string from the picture, cut the string into two pieces of different lengths and throw one piece away. Then I attach the ends of the remaining piece to *A* and *B* again. When the picture is now hung on the nail the top of the picture is 7 cm higher than it was before. How long was the piece of string that I threw away?   [3]

11   *Answer this question on the insert sheet provided.*

TURN OVER!

12   a) The following four positive whole numbers are written in increasing numerical order.

$$w \quad x \quad y \quad z$$

The median of the numbers is 7.
The mode of the numbers is 7.
The mean of the numbers is 6.

Find the possible values of w. [4]

    b) The following five positive whole numbers are written in increasing numerical order.
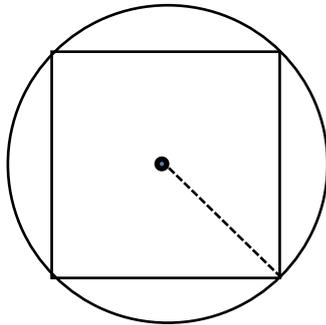
$$a \quad b \quad c \quad d \quad e$$

The mode of the numbers is 7.
The mean of the numbers is 4.
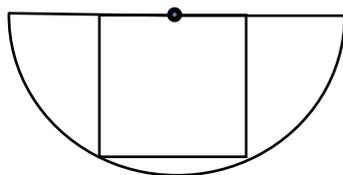
Find the possible values of a. [3]

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13   a) A square is inscribed in a circle of radius r as shown below. Find the area of the square in terms of r. [3]


    b) Another square is inscribed in a semicircle also of radius r. Find the area of the smaller square in terms of r. [3]

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14   *A*, *B*, *C* and *D* are connected by the formula $A = \dfrac{B \times C}{D}$. If *B* is increased by

32%, *C* is decreased by 65% and *D* is decreased by 45%, determine the
percentage change in *A*, stating whether it is an increase or a decrease.   [4]

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15   In this diagram all angles are measured in degrees. If *T* = 12 find the value of *x*.   [4]

$3x + 3y$

$3T$

$6x - 2y$

$6y + 54$

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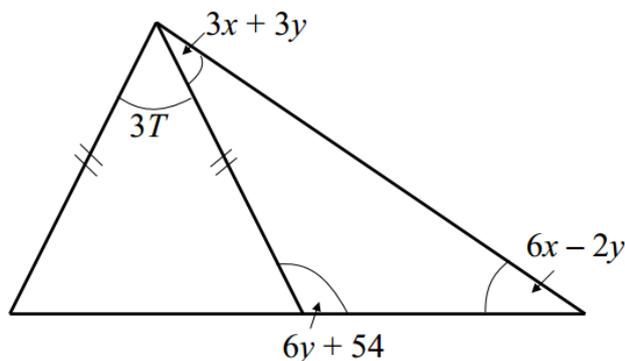
16   A certain sink has three taps: hot, cold, and drinking.
The cold tap on its own can fill the sink in 2 minutes.
The hot tap on its own can fill the sink in 3 minutes.
The drinking tap on its own can fill the sink in 5 minutes. How long will it take
to fill the sink if

    a)   the hot and cold taps are both running but the drinking tap is not?   [3]

    b)   all three taps are running?   [2]

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17   The symbol *n*! means $n \times (n - 1) \times (n - 2) \times \ldots \times 3 \times 2 \times 1$.
Thus $3! = 3 \times 2 \times 1 = 6$.
Calculate the values of:

    a)   5!   [1]

    b) $\dfrac{9!}{7!}$ [2]

    c) $\dfrac{m!}{(m-1)!}$ [2]

    d)   Show that, for any value of *a*, $a! + (a - 1)! = (a + 1) \times (a - 1)!$.   [2]

    Simplify as far as possible:

    e) $\dfrac{m! - (m-1)!}{m! + (m-1)!}$ [2]

    f) $\dfrac{32! - 31!}{32! + 31!} \times \dfrac{30! - 29!}{30! + 29!} \times \dfrac{28! - 27!}{28! + 27!}$ [3]

TURN OVER!

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1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
		93	94	95	96	97	98	99	100
				105	106	107	108	109	110
					116	117	118	119	1
					127	128			

The grid above has 10 columns and an infinite number of rows.
On the grid is a *T* shape.
The number at the bottom of the *T* is called the *T*-number, n, so that for this position of the *T*-shape, $n = 45$.
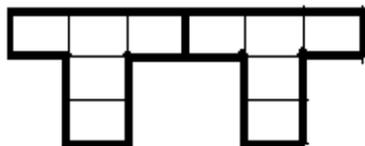The sum of values within the *T* shape is called the *T*-sum, S_n, so that for this position of the *T*-shape,

$$S_{45} = 24 + 25 + 26 + 35 + 45 = 155.$$

a) Find the value of:

   i) S_{28} [1]

   ii) S_{203} [1]

b) Find a formula for S_n in terms of n. [1]

c) State, giving reasons, whether the following values of S_n are possible:

   i) $S_n = 190$ [1]

   ii) $S_n = 146$ [1]

   iii) $S_n = 230$ [1]

The grid changes so that, instead of 10 columns, there are now m columns. (The numbers still run consecutively.)

d) Find a formula for S_n on the new grid, in terms of n and m. [2]

Two *T*-shapes joined side by make a *pi-shape*:

The *pi-number* k is equal to the *T*-number n of the left-hand *T*, and the *pi-sum* P_n is equal to the sum of all ten values within the pi-shape.

e) Given that the pi-sum P_n of a certain pi-shape is 99, find the smallest possible number of columns the grid can have. [4]

END OF EXAMINATION   *[Have you answered Question 11?]*

END OF PAPER

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