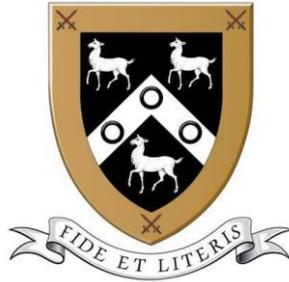


FULL NAME

PRESENT SCHOOL



**ST PAUL'S SCHOOL
JUNIOR SCHOLARSHIP EXAMINATION**

MAY 2016

MATHEMATICS

2 hours

Write all your answers and working on this question paper.

You may use rough paper if you wish but this should not be handed in.

Answer as many questions as you can in any order you wish.

Credit will be given for reasoning and working where appropriate.

When the answer is a fraction it should be given in mixed form, e.g. $3\frac{4}{5}$

The total number of marks for this paper is 100.

The mark allocation is shown in brackets at the end of each part of each question.

Please write your full name and school at the top of this page.

CALCULATORS MAY NOT BE USED.

1 Find the value of each of the following when $a = 1, b = -2, c = 3$:

(i) $ab(a + b + c)^2$,

Answer [2]

(ii) $(ac - 2b^2)^c$.

Answer [2]

2 Expand and simplify fully:

(i) $(3x + 2)^2$,

Answer [2]

(ii) $(2x - 1)(4x + 2)$,

Answer [2]

(iii) $x^2(2x^2 - 1)(4x^2 + 2)$.

Answer [2]

3 The numbers A and B are written in prime factors as

$$A = p^{10} q^{12} r^5 s^{101}$$

$$B = p^2 q^{31} r^9 s^{16} t^7.$$

Write down the LCM and the HCF of A and B in terms of p, q, r, s and t .

$$LCM = \dots\dots\dots$$

$$HCF = \dots\dots\dots [2]$$

4 Solve the following equations:

(a) $2(x - 1) - 3(4 - 2x) = 4$

$$x = \dots\dots\dots [3]$$

(b) $\frac{1}{3}(\frac{2}{3}y - 1) = \frac{1}{2}(2 - 3y)$

$$y = \dots\dots\dots [3]$$

(c) $\frac{2}{1-z} = \frac{3}{1+z}$.

$$z = \dots\dots\dots [3]$$

- 5 A fair six-sided die is thrown twice and the scores obtained are multiplied together. What is the probability that the resulting number is square? Show all your working.

Answer [4]

- 6 For numbers p and q we define $p * q$ to be the power to which we must take p in order to get q . For example:

$$2 * 8 = 3$$

because $2^3 = 8$, and

$$\frac{1}{2} * \frac{1}{4} = 2$$

because $(\frac{1}{2})^2 = \frac{1}{4}$, and so on.

- (i) Find the value of $3 * 81$.

Answer [1]

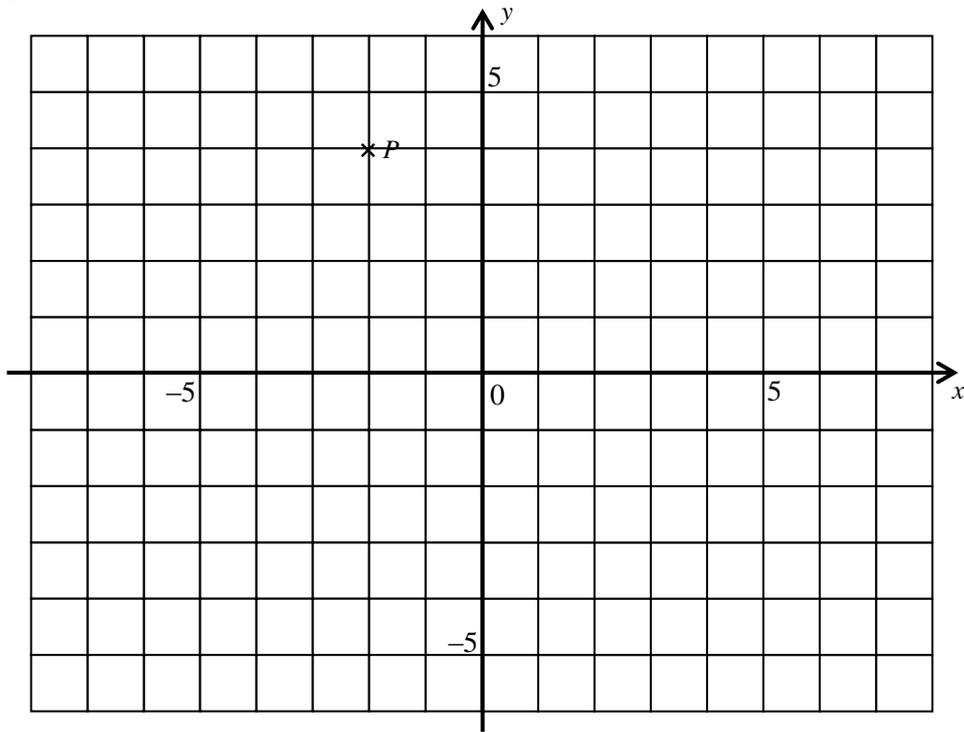
- (ii) Find the value of $2016 * 2016^{2016}$.

Answer [1]

- (iii) Solve the equation $2 * x = 5$.

Answer [2]

- 7 The point P has coordinates $(-2, 4)$.



- (i) P is translated by 1 unit to the left and 3 units down to give the point Q . Write down the coordinates of Q .

Answer (.....,) [1]

- (ii) Q is reflected in the line $y = x$ to give the point R . Write down the coordinates of R .

Answer (.....,) [2]

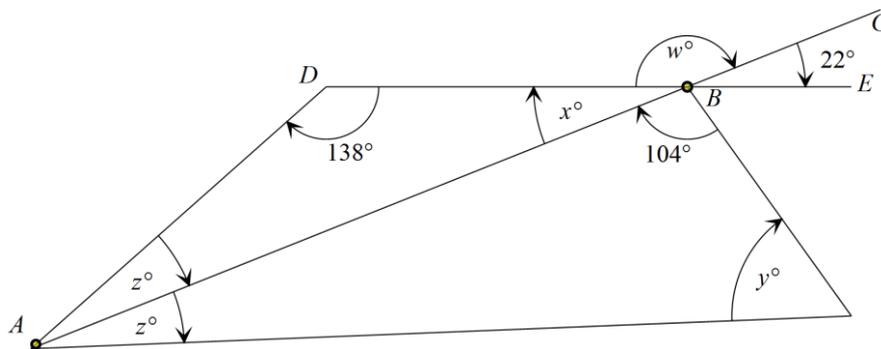
- (iii) R is rotated through 90° anticlockwise about $(0, 0)$ to give the point S . Write down the coordinates of S .

Answer (.....,) [2]

- (iv) Describe the single translation that takes P to S .

Answer [1]

- 8 Find the sizes of the angles w , x , y , z in the diagram below. ABC and DBE are straight lines.



$$w = \dots\dots\dots^\circ \qquad x = \dots\dots\dots^\circ$$

$$y = \dots\dots\dots^\circ \qquad z = \dots\dots\dots^\circ \quad [4]$$

- 9 (a) Cameron is driving his car at 70 miles per hour. His friend Christian is driving his car at 60 miles per hour and is currently 3 miles ahead. How long, in minutes, will it take Cameron to catch up with Christian?

Answer minutes [3]

- (b) After all that driving Cameron takes his car to his local car wash. At the car wash three men can wash four cars in 20 minutes. How many minutes will it take one man to wash Cameron's car?

Answer minutes [3]

- 10 (a) The number P is increased by 10% to give the number Q . The number Q is decreased by 10% to give the number R . Explain whether R is greater than P , less than P , or equal to P .

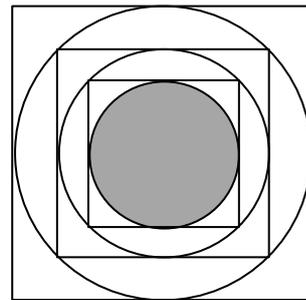
Answer

..... [3]

- (b) A number is increased by 20% and then decreased by $x\%$. The resulting number is 10% less than the number we started with. Find the value of x .

$x = \dots\dots\dots$ [3]

- 11 The diagram below shows a series of circles and squares. Each shape just touches the shape inside it and the shape outside it. The outer square has sides of length 2 cm. Find the area of the shaded region, showing all your working. Leave π in your answer.



Area cm^2 [5]

12 Thirty students sat a mathematics test marked out of 75. Their average mark is 47.0.

- (i) It is found that one of the marks was recorded incorrectly. What are the **maximum** and **minimum** possible average marks after the incorrect mark has been corrected?

Maximum = *minimum* = [2]

- (ii) Once the incorrect mark has been corrected the average mark is 49.0. What is the minimum possible value of the corrected mark?

Answer [2]

- (iii) It is now found that the twos and sixes in the marks have been interchanged: that is, whenever a two appeared it was recorded as a six and vice-versa. So, for example, a mark of 52 was recorded as 56 while a mark of 61 would have been recorded as 21, and so on. After investigation it was found that this issue affected six of the thirty marks. What is the biggest possible change in the average mark as a result of this issue?

Answer [3]

- 13 (i) Solve the simultaneous equations $3x + 2y = 30$, $y - x = 5$.

$$x = \dots\dots\dots y = \dots\dots\dots [3]$$

Use your answer to part (i) to solve the following equations. *[Answers that do not make use of part (i) will gain no credit.]*

(ii) $\frac{3}{p} + \frac{2}{q} = 30$, $\frac{1}{q} - \frac{1}{p} = 5$,

$$p = \dots\dots\dots q = \dots\dots\dots [2]$$

(iii) $3w^2 + 2t^2 = 30$, $t^2 - w^2 = 5$.

$$w = \dots\dots\dots t = \dots\dots\dots [2]$$

14 The first term of a sequence of numbers is 1. Other terms are given by the rule:

$$\boxed{\text{next term} = 2 \times (\text{present term}) + 1} \quad (*)$$

- (i) Find the second, third and fourth terms of the series, and show that the fifth term is 31.

Second term *Third term* *Fourth term*

Fifth term:

[4]

- (ii) Guess a formula for the n^{th} term of the series, in terms of n .

Answer [2]

- (iii) Explain why all the numbers in the sequence are odd.

Answer

[2]

- (iv) Now consider a new sequence consisting of the first, third, fifth, seventh, ... terms of the original sequence. Find a rule connecting the terms of this new sequence. (Your rule should look like the original rule, (*), with different numbers).

Answer: Next term = [2]

- (v) Every term in this new sequence (apart from the first) gives the same remainder when it is divided by 4. Use your answer to part (iv) to state what this remainder is, with a reason.

Answer

[2]

15 The price of a car, in pounds, is shown on four cards, on each of which is written a single figure between 0 and 9. The first card shows the number of thousands, the second the number of hundreds and so on. On a windy day the card showing the thousands digit blows away and the price now displayed on the remaining three cards is $\frac{1}{49}$ of the intended price.

- (i) Let a denote the number shown on the thousands card, b the number shown on the hundreds card, c the number shown on the tens card and d the number shown on the units card so that the price of the car is $1000a + 100b + 10c + d$. Show that

$$d = \frac{125}{6}a - 100b - 10c.$$

[3]

- (ii) Explain why a must be 6.

Answer

..... [2]

- (iii) Find the values of b , c and d , and hence state the intended full price of the car.

Answer: $b = \dots, c = \dots, d = \dots, \text{intended price} = \text{£}\dots$ [4]

16 The sum of the first n cubic numbers is given by

$$1^3 + 2^3 + 3^3 + \dots + n^3 = an^2(n + b)^2,$$

where a and b are fixed positive numbers.

(a) By substituting suitable values of n , show that $a(1 + b)^2 = 1$ and $4a(2 + b)^2 = 9$.

[3]

(b) Show that $\frac{2+b}{1+b} = \frac{3}{2}$, and hence determine the value of b and the value of a .

Answer: $b = \dots\dots\dots a = \dots\dots\dots$ [4]

(c) Use your formula to calculate the value of $1^3 + 2^3 + 3^3 + \dots + 100^3$.

Answer $\dots\dots\dots$ [2]

END OF EXAMINATION